# EE 508

## Lecture 35

## Transconductor Design

#### Transconductor Design Review from Last Lecture



Transconductor-based filters depend directly on the  $g<sub>m</sub>$  of the transconductor

Feedback is not used to make the filter performance insensitive to the transconductance gain

Linearity and spectral performance of the filter strongly dependent upon the linearity of the transconductor

Often can not justify elegant linearization strategies in the transconductors because of speed, area, and power penalties

# Linearity of Amplifiers



Strongly dependent upon linearity of transconductance of differential pair

# Linearity of Amplifiers



# Differential Input Pairs





MOS Differential Pair **Bipolar Differential Pair** 

#### Signal Swing and Linearity Review from Last Lecture



# Linearity of Amplifiers



Strongly dependent upon linearity of transconductance of differential pair

#### How linear is the amplifier ? **Review from Last Lecture**



#### How linear is the amplifier ? **Review from Last Lecture**



It can be shown that the deviation from the line in % is given by

$$
\theta = 100\% \left( 1 - \sqrt{1 - \frac{\left( V_d \sqrt{V_{EB}} \right)^2}{4}} \right)
$$



#### What swings on drain currents are typical when using the differential pair in a voltage amplifier (Op Amp)? Review from Last Lecture



$$
V_{\text{INpp}} = 0.05 V_{\text{EB1}}
$$

 $V_d$ 

 $\sqrt{2}V_{EB1}$ 

 $V<sub>EB1</sub>$ 

1% Linear <sup>=</sup>  $0.5V<sub>FB1</sub>$ 

 $_{\rm I}$ 

ID1

- This results in a very small nonlinearity in the Op Amp even with very large swings on the output.
- The current change is also very small
- When used in two-stage voltage amplifier structure, the nonlinearity in this structure is even much smaller!

#### What swings on drain currents are typical when using the differential pair in a voltage amplifier (Op Amp)? Review from Last Lecture





If  $λ = 01V^{-1}$ 

$$
V_{\textsf{INpp}} = 0.05 V_{\textsf{EB1}}
$$

- This results in a very small nonlinearity in the Op Amp even with very large swings on the output.
- The current change is also very small
- When used in two-stage voltage amplifier structure, the nonlinearity in this structure is even much smaller!

Does this imply that large swings on the output introduce very little nonlinearity when used as an OTA?

No ! Because when used as an OTA the voltage swings in the input and output are often about the same!

#### Programmable Filter Structures



$$
\big|\varpi_0\big|=\frac{\mathcal{G}_m}{C}
$$

Often want to program or trim filters (i.e. trim  $\omega_0$ )

Applicable in wide variety of filter architectures (here showing integrator-based)

Attractive to do this by adjusting  $g_m$ , in part, because  $g_m$  can be continuously adjustable with some transconductance devices

What input range is possible when using the tail current to program the OTA (i.e. after W/L fixed)? Review from Last Lecture





$$
g_m = \mu C_{ox} \frac{W}{L} V_{EB} = \sqrt{I_T} \sqrt{\mu C_{ox} \frac{W}{L}}
$$
\n
$$
V_{dx} = \pm \sqrt{\frac{2L}{\mu C_{ox} W}} (\sqrt{I_T})
$$

- Input signal swing decreases linearly with decreases in  $g_m$  for fixed W/L
- One decade reduction in  $g_m$  results in one decade decrease in signal swing
- One decade reduction in  $g_m$  requires two decade decrease in  $I_T$
- Though MOS OTA can have very good single swing with large  $V_{FB}$ , very limited tail current programmability with basic MOS OTA  $g_m = \mu C_{ox} \frac{W}{L} V_{EB} = \sqrt{l_{\tau}} \sqrt{\mu C_{ox} \frac{W}{L}}$ <br>
Input signal swing decreases linearly with decreases in  $g_m$  for fixed W/L<br>
One decade reduction in  $g_m$  results in one decade decrease in signal swin<br>
One decade reduction i
- There are, however, other ways to program MOS OTA without big penalty in

### Bipolar Differential Pair



$$
I_{\mathsf{d}} = V_t \Bigg( \text{In} \Bigg( \frac{I_{\mathsf{C2}}}{J_{\mathsf{S}} \boldsymbol{\mathsf{A}}_{\mathsf{E2}}} \Bigg) - \text{In} \Bigg( \frac{I_{\mathsf{C1}}}{J_{\mathsf{S}} \boldsymbol{\mathsf{A}}_{\mathsf{E1}}} \Bigg) \Bigg) \bigg.^{\boldsymbol{\mathsf{A}}_{\mathsf{E1}} = \boldsymbol{\mathsf{A}}_{\mathsf{E2}}} V_t \text{In} \Bigg( \frac{I_{\mathsf{C2}}}{I_{\mathsf{C1}}} \Bigg)
$$

### Bipolar Differential Pair



At  $I_{C1} = I_{C2} = I_T/2$ ,  $V_d = 0$ 

As  $I_{C1}$  approaches 0,  $V_{d}$  approaches infinity

As l<sub>C1</sub> approaches I<sub>T</sub>, V<sub>d</sub> approaches minus infinity Transition much steeper than for MOS case

#### Transfer Characteristics of Bipolar Differential Pair



Transition much steeper than for MOS case Asymptotic Convergence to 0 and  $I<sub>T</sub>$ 

### Signal Swing and Linearity of Bipolar Differential Pair



Note  $\rm V_{dint}$  is independent of I $_{\rm T}$  in contrast to what we saw for MOS differential pairs

### Signal Swing and Linearity of Bipolar Differential Pair



### Signal Swing and Linearity of Bipolar Differential Pair



What input range is possible when using the tail current to program the OTA ?





Since input signal swing not affected by I<sub>T</sub>, Multi-decade adjustment of  $\mathsf{g}_{\mathsf{m}}$ with  $I_T$  can be made without degrading signal swing

#### Signal Swing for basic MOS and BJT transconductors



# Signal Swing and Linearity Summary

- Signal swing of MOSFET can be rather large if  $V_{FB}$  is large but this limits gain
- Signal swing of MOSFET degrades significantly if  $V_{FB}$  is changed for fixed W/L
- Bipolar swing is very small but independent of  $g_m$
- Multiple-decade adjustment of bipolar  $g_m$  is practical
- Even though bipolar input swing is small, since gain is often very large, this small swing does usually not limit performance in feedback applications when used as a voltage amplifier

### Does the MOS or BJT transconductor have larger input signal swing?



Depends upon how much adjustment range is desired





$$
I_0 = I_1 - I_2
$$
  
\n
$$
I_1 = \beta_1 (V_G - V_X - V_{Tn})^2
$$
  
\n
$$
I_1 = \beta_2 (V_X - V_{in} + V_{Tp})^2
$$
  
\n
$$
I_2 = \beta_3 (V_{in} - V_Y - V_{Tn})^2
$$
  
\n
$$
I_2 = \beta_4 (V_Y + V_{G1} + V_{Tp})^2
$$

Taking the square root of the two  $I_1$  equations

$$
\sqrt{\frac{1}{\beta_1}}\sqrt{I_1} = (V_G - V_X - V_{Tn})
$$

$$
\sqrt{\frac{1}{\beta_2}}\sqrt{I_1} = (V_X - V_{in} + V_{Tp})
$$

Adding these two equations, we obtain

$$
\left(\sqrt{\frac{1}{\beta_2}} + \sqrt{\frac{1}{\beta_1}}\right)\sqrt{I_1} = \left(V_G - V_{in} + V_{Tp} - V_{Tn}\right)
$$
  
Similary, for the last two equations, obtain  

$$
\left(\sqrt{\frac{1}{\beta_3}} + \sqrt{\frac{1}{\beta_4}}\right)\sqrt{I_2} = \left(V_{G1} + V_{in} + V_{Tp} - V_{Tn}\right)
$$



 $I_0 = I_1 - I_2$  $\overline{N_1} = ( V_G - V_{in} + V_{Tp} - V_{Tn} )$ 2  $V$   $P_1$  $\frac{1}{1}$  +  $\frac{1}{2}$   $\sqrt{I_1}$  =  $(V_C - V_{in} + V_{in} - V_{in}$  $\beta_2 \quad \sqrt{\beta_1}$  $(\sqrt{1} \sqrt{1})$  $\left[\frac{1}{2} + \frac{1}{2} - \frac{1}{2} \right] \sqrt{I_1} = \left(V_G - V_{in} + V_{To} - \frac{1}{2}\right)$  $(\bigvee \beta_2 \quad \bigvee \beta_1)$  $\overline{C}_2 = ( V_{G1} + V_{in} + V_{Tp} - V_{Tn} )$ 3  $V$   $\mu$  $\frac{1}{1}$  +  $\frac{1}{1}$   $\sqrt{I_2}$  =  $(V_{C1} + V_{in} + V_{T2} - V_{in}$  $\beta_{3}$   $\bigvee \beta_{4}$  $(\sqrt{1} \sqrt{1})$  $\int_{\alpha}^{1} \frac{1}{\sqrt{2}} + \int_{\alpha}^{1} \frac{1}{\sqrt{2}} = (V_{G1} + V_{in} + V_{To})$  $(\bigvee \beta_3 \quad \bigvee \beta_4)$ 

Squaring the last two equations we obtain

$$
I_1 = \beta_5 (V_G - V_{in} + V_{Tp} - V_{Tn})^2
$$
  

$$
I_2 = \beta_6 (V_{G1} + V_{in} + V_{Tp} - V_{Tn})^2
$$

Define



Equating the difference to  $I_0$ , we obtain  $(\beta_{\mathsf{5}}\hspace{-0.09cm}-\hspace{-0.09cm}\beta_{\mathsf{6}}) \mathrm{V}_{\mathrm{in}}^2$  $+ V_{in} \left( 2 \beta_5 \left[ V_{Tn} - V_{Tp} - V_G \right] + 2 \beta_6 \left[ V_{Tn} - V_{Tp} + V_{G1} \right] \right)$  $I_0 = (\beta_5 - \beta_6) V_{in}^2$ 2  $\Gamma$   $\Gamma$  $+ \beta_{5} \left[ V_{\text{Tp}} - V_{\text{Tn}} + V_{\text{G}} \right]^{2} - \beta_{6} \left[ V_{\text{Tp}} - V_{\text{Tn}} + V_{\text{G1}} \right]$ 



$$
I_0 = (\beta_5 - \beta_6) V_{in}^2
$$
  
+  $V_{in} \left( 2\beta_5 \left[ V_{Tn} - V_{Tp} - V_G \right] + 2\beta_6 \left[ V_{Tn} - V_{Tp} + V_{G1} \right] \right)$   
+  $\beta_5 \left[ V_{Tp} - V_{Tn} + V_G \right]^2 - \beta_6 \left[ V_{Tp} - V_{Tn} + V_{G1} \right]^2$ 

If size devices so that  $\beta_5 = \beta_6$  and  $V_G = V_{G1}$ , this simplifies to

$$
I_0 = V_{in} \left( 4\beta_5 \left[ V_{Tn} - V_{Tp} - V_G \right] \right)
$$

Note this behaves as a linear transconductor !

$$
g_m = 4\beta_5 \Big[ V_{Tn} - V_{Tp} - V_G \Big]
$$

- Since both  $M_2$  and  $M_3$  are driven, this is a power-efficient method for generating a given  $g_m$
- Behavior will degrade with bulk-dependent threshold voltages of n-channel devices
- Would like to generate  $\mathsf{V}_{\mathsf{G}}$  and  $\mathsf{V}_{\mathsf{G}1}$  independent of  $\mathsf{V}_{\mathsf{DD}}$

# Bias Generators

Bias voltage generators are widely used to bias cascode devices and other transistors in an IC

Key goal is often to have bias voltages independent of  $V_{DD}$  to avoid coupling supply noise into linear circuits

#### Potential Bias Generators

Consider the following four circuits:



#### Potential Bias Generators



- If  $g_{\text{o}}$  is neglected, it can be shown that all devices are operating in the saturation region, the output voltages are independent of  $V_{DD}$
- Note all have a positive feedback loop !

Regenerative Feedback Loops Can Provide Some Very Useful Properties but Can Also Offer Some Surprises !!

Theorem: If the small signal loop gain of the positive feedback loop is less than unity at an equilibrium point of the return map, then the equilibrium point is a stable equilibrium point and if the loop gain is larger than unity at an equilibrium point the equilibrium point is an unstable equilibrium point.



## Consider the Inverse Widlar Bias Generator



Can be viewed as two common-source amplifiers in a loop



Same observation about the other 3 structures

# $V_{DD}$  Independent Bias Generators

Consider the two Inverse Widlar bias generators (start-up ckts not shown)





 $01 - v_{\text{Th}}$ 

 $02 - v_{\text{Th}}$ 

=

2

=

 $V_{01} = V_{T_{n}}$   $\frac{1}{\sqrt{24+12}}$ 

 $V_{02} = V_{T_{\rm m}} \left[ \frac{1}{\sqrt{2\pi}} \frac{3}{2} \frac{2}{\sqrt{2\pi}} \frac{1}{\sqrt{2\pi}} \frac{34}{\sqrt{2\pi}} \frac{1}{2} \right]$ 

 $2 - 1$ 

 $\begin{pmatrix} 0 & \sqrt{N_2} & \sqrt{N_3} & \sqrt{N_4} & \sqrt{N_4} & \sqrt{N_5} & \sqrt{N_6} & \sqrt{$  $1-\sqrt{\frac{W_2H_1}{M_1+M_2}}$  |

+ <sup>−</sup>  $\left(\begin{array}{cc} \sqrt{W_3L_2} & \sqrt{M_{54}W_1L_2}\end{array}\right)$ 

 $1 - \frac{W_2 L_1}{W_2 L_1}$ 

 $1 + \sqrt{\frac{W_2 L_3}{\omega}} - \sqrt{\frac{W_2 L_1}{\omega}}$ 

 $M_{54}W_1L_2$  |

 $W_2L_2$   $\sqrt{M_{54}W_1L_2}$ 

 $1 + \frac{W_2 L_3}{W_2 L_1}$ 

 $1 + \frac{W_2 L_3}{W_2 L_1}$  |

 $W_2L_2$   $\sqrt{M_{54}W_1L_2}$ 

 $W_2L_2$   $\sqrt{M_{54}W_1L_2}$   $\sqrt{M_{15}W_2}$ 

 $54 \text{ } \text{V}1 \text{ } -2 \text{ }$ 

 $3\frac{1}{2}$   $\sqrt{1154}$   $\sqrt{112}$   $\sqrt{112}$ 

 $2\frac{1}{3}$   $2\frac{1}{1}$   $1\frac{1}{2}\frac{1}{2}$  $3\text{--}2$   $\sqrt{1133} \sqrt{1132}$ 

 $\left(\begin{array}{cc} \overline{\text{W}_{2} \text{L}_{2}} & \overline{\text{W}_{2} \text{L}_{4}} \end{array}\right)$  $1 + \sqrt{\frac{1223}{Nkt}} - 2\sqrt{\frac{1221}{Nkt}}$ 

> $2^{\mathsf{L}}3$  1  ${\bf w}_2{\bf L}_1$  1  $3\text{--}2$   $\sqrt{1133} \sqrt{1132}$

+  $\left| \frac{2}{2} - \right| - \frac{2}{2} - \left| \frac{2}{2} - \frac{1}{2} \right|$ 

 $2^{\mathsf{L}}3$  1  ${\bf w}_2{\bf L}_1$  1

Assuming all devices in saturation at desired operating point,

$$
V_{02} = V_{Tn} + \frac{\theta}{2} \pm \sqrt{\frac{\theta V_{Tn}}{2} + \left(\frac{\theta}{2}\right)^2}
$$
  
\n
$$
V_{01} = V_{Tn} \left( \frac{V_{01}}{1 + \sqrt{\frac{W_{2}L_{3}}{W_{3}L_{2}}} - \sqrt{\frac{2L_{2}}{\mu_{n}C_{OX}W_{2}R_{1}}} \sqrt{V_{Tn} + \frac{\theta}{2} \pm \sqrt{\frac{\theta V_{Tn}}{2} + \left(\frac{\theta}{2}\right)^2}}
$$
  
\n
$$
V_{02} = V_{Tn} \left( \frac{1 + \sqrt{\frac{W_{2}L_{3}}{W_{3}L_{2}}} - \sqrt{\frac{2L_{2}}{\mu_{n}C_{OX}W_{2}R_{1}}} \sqrt{V_{Tn} + \frac{\theta}{2} \pm \sqrt{\frac{\theta V_{Tn}}{2} + \left(\frac{\theta}{2}\right)^2}}
$$
  
\n
$$
V_{02} = V_{Tn} \left( \frac{1 + \sqrt{\frac{W_{2}L_{3}}{W_{3}L_{2}}} - \sqrt{\frac{2L_{2}}{\mu_{n}C_{OX}W_{2}R_{1}}} \sqrt{\frac{2L_{3}}{W_{3}C_{X}}} - \sqrt{\frac{2L_{3}}{\mu_{n}C_{OX}W_{2}R_{1}}} \sqrt{\frac{2L_{3}}{W_{3}C_{X}}} - \sqrt{\frac{2L_{3}}{\mu_{n}C_{OX}W_{2}
$$

where  $\theta = \frac{2L_1}{M_{54}R_1\mu_nC_{OX}W_1}$  and  $M_{54}$  is the  $M_5$ : $M_4$  mirror gain  $\sqrt{W_3L_2} W_{54}W_1L_2$  $1$  $54N_1\mu_p$   $\sqrt{OX}$   $N_1$  and  $N_54$  is the  $N_5$ .  $2L<sub>1</sub>$  $M_{\rm 54}R_{\rm 1}\mu_{\rm n}C_{\rm OV}W_{\rm 1}$  and M $_{\rm 54}$  is the M $_{\rm 5}$ :M $_{\rm 4}$  mirror gain  $\theta =$  -  $\theta$  -  $\theta$  -  $\theta$  $\mu_{\sf n}$ C<sub>OX</sub>W<sub>1</sub> and M<sub>54</sub> is the M<sub>5</sub>.M = Note: Outputs  $V_{DD}$  independent !

# $V_{DD}$  Independent Bias Generators

Consider the two Inverse Widlar bias generators (start-up ckts not shown)



Must still check for stationarity of operating point, stability, and start-up

#### Consider Inverse Widlar with Transistor  $\mathsf{M}_3$  first





# $V_{DD}$  Independent Bias Generators

Check for stationarity of operating point  $V_{DD}$ 





For any operating point when circuit designed for all devices operating in saturation:

$$
A_{Loop} = \frac{g_{m1}}{g_{m5}} \cdot g_{m4} \left( \frac{1}{g_{m2}} + \frac{1}{g_{m3}} \right)
$$

$$
g_{m} = \frac{2I_{Q}}{V_{EB}}
$$

$$
A_{Loop} = \frac{V_{EB2} + V_{EB3}}{V_{EB2} + V_{EB3} + V_{Tn}}
$$

- Observe loop gain is always less than 1
- So it is a viable circuit for a bias generator

## $V_{DD}$  Independent Bias Generators Check for stability



- Circuit has 3 poles
- May use RH criteria
- If unstable, adjust one of the capacitors

# $V_{DD}$  Independent Bias Generators



Must have single intersection point (desired point) with slope at unity gain crossing less than 1 over PVT variations Add/modify startup circuit if necessary (usually necessary with this structure)<br>Add/modify startup circuit if necessary (usually necessary with this structure)

#### Consider Inverse Widlar with Transistor Resistor





# $V_{DD}$  Independent Bias Generators

Check for stationarity of operating point



- Observe loop gain is always larger than 1
- So it is not a viable circuit for a bias generator

# Basic Bias Generator Circuits

Only two of these circuits are useful directly as bias generators!







$$
I_{D1} = \beta (V_1 - V_{S1} - V_T)^2
$$
  
\n
$$
I_{D2} = \beta (V_2 - V_{S2} - V_T)^2
$$
  
\n
$$
V_{S1} - I_{D1}R_{S1} = V_{S2} - I_{D2}R_{S2}
$$
  
\n
$$
I_{D1} + I_{D2} = I_T
$$

With a straightforward analysis, we obtain the expression

$$
V_{d} = \sqrt{\frac{1}{\beta}} \left( \sqrt{I_{T} - I_{D1}} - \sqrt{I_{D1}} \right) + R_{S} \left( I_{T} - 2I_{D1} \right)
$$

The first term on the right is the nonlinear term of the original source coupled pair and the second is linear in  $I_{D1}$ 

The larger the second term becomes, the more linear the transfer characteristics are



$$
\sqrt{\frac{1}{\beta}} \left( \sqrt{I_T - I_{D1}} - \sqrt{I_{D1}} \right) + R_S (I_T - 2I_{D1}) = V_d
$$

The transconductance of this structure can be readily derived to obtain

$$
g_{m} = \frac{\partial V_{d}}{\partial I_{D1}}\Big|_{Q-pt}^{-1} = \left[\sqrt{\frac{1}{\beta}} \cdot \frac{1}{2} \left(-\left(I_{T} - I_{D1}\right)^{-\frac{1}{2}} - I_{D1}^{-\frac{1}{2}}\right) - 2R_{S}\right]\Big|_{Q-pt}^{-1}
$$

This can be expressed as

$$
g_{\rm m} = \frac{\partial V_{\rm d}}{\partial I_{\rm D1}}\Big|_{Q-\rm pt}^{-1} = -\frac{1}{\left[\sqrt{\frac{2}{\beta I_{\rm T}}} + 2R_{\rm S}\right]} = -\frac{\beta V_{\rm EB}}{1 + 2\beta V_{\rm EB}R_{\rm S}}
$$



There are a host of transconductance linearization strategies that have been discussed in the literature

Some are shown below

Many are strongly dependent upon a precise square-law model of the MOS devices and do not provide practical solutions when the devices are not square-law devices

Analysis or simulation with a more realistic model is necessary to validate linearity and practical applications of these structures

How good is the square-law model that we have been using for predicting filter performance?

It is reasonably good when analyzing structures whose linearity characteristics are not strongly dependent upon the device model

The circuits considered to date are not particularly linear so the square-law model probably does a pretty good job of predicting their performance

More accurate models are usually unwieldy for hand analysis



Fig. 1 Linearised CMOS transconductance circuit

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From CAS 2006 P 811 Jose Silva



Linearity Enhancement with Source Degeneration





Linearization with active source degeneration

#### **CMOS transconductance amplifiers, architectures and** active filters: a tutorial

E.Sánchez-Sinencio and J.Silva-Martínez

Abstract: An updated version of a 1985 tutorial paper on active filters using operational transconductance amplifiers (OTAs) is presented. The integrated circuit issues involved in active filters (using CMOS transconductance amplifiers) and the progress in this field in the last 15 years is addressed. CMOS transconductance amplifiers, nonlinearised and linearised, as well as frequency limitations and dynamic range considerations are reviewed. OTA-C filter architectures, current-mode filters, and other potential applications of transconductance amplifiers are discussed.



Linearity compensation with cross-coupled feedback

## Single-ended input TAs



## Differential input OTAs





Differential input and output OTAs

## Parasitic Capacitances and Floating Nodes



Recall: A floating node is a node that is not connected to either a zeroimpedance element or across a null-port

Floating nodes are generally avoided in integrated filters because the parasitic capacitances on the floating nodes usually degrades filter performance and often increases the order of the filter

Some filter architectures inherently have no floating nodes, specifically, most of the basic integrator-based active RC filters have no floating nodes

Invariably the OTA-C integrators have floating nodes so are sensitive to parasitic capacitances

When filters are programmable or calibrated, floating nodes are less problematic but may add nonlinearity

# Signal Swing in OTA Circuits

The signal swing for the basic bipolar OTA is limited to a few mV for reasonably linear operation

This limited signal swing limits the use of the OTA

The following circuit (with maybe a 100:1 or more attenuation) can be used to increase the input signal swing to the volt range and although it involves quite a few more components, the functionality can be most significant

Program range is not affected by adding the attenuators



R. L. Geiger and E. Sánchez-Sinencio, "Active Filter Design Using Operational Transconductance Amplifiers: A Tutorial," IEEE Circuits and Devices Magazine, Vol. 1, pp.20-32, March 1985.

### **Active Filter Design Using Operational** Transconductance Amplifiers: A Tutorial

Randall L. Geiger and Edgar Sánchez-Sinencio

## Programmable Filter Structures

It will be assumed that the transconductance gain can be programmed with either a dc current or a dc voltage



## Programmable Filter Structures



Programmable First-Order High-Pass Filter





# Stay Safe and Stay Healthy !

# End of Lecture 35